Chapter 6

LINEAR INEQUALITIES

6.1 Overview

6.1.1 A statement involving the symbols '>', '<', '≥', '≤' is called an inequality. For example $5 > 3$, $x \le 4$, $x + y \ge 9$.

(i) Inequalities which do not involve variables are called numerical inequalities. For example $3 < 8$, $5 \ge 2$.

(ii) Inequalities which involve variables are called literal inequalities. For example, $x > 3$, $y \le 5$, $x - y \ge 0$.

(iii) An inequality may contain more than one variable and it can be linear, quadratic or cubic etc. For eaxmple, $3x - 2 < 0$ is a linear inequality in one variable, $2x + 3y \ge 4$ is a linear inequality in two variables and $x^2 + 3x + 2 < 0$ is a quadratic inequality in one variable.

(iv) Inequalities involving the symbol '>' or '<' are called **strict inequalities**. For example, $3x - y > 5$, $x < 3$.

(v) Inequalities involving the symbol '≥' or '≤' are called **slack inequalities**. For example, $3x - y \ge 5$, $x \le 5$.

6.1.2 *Solution of an inequality*

(i) The value(s) of the variable(s) which makes the inequality a true statement is called its **solutions**. The set of all solutions of an inequality is called the **solution set** of the inequality. For example, $x - 1 \ge 0$, has infinite number of solutions as all real values greater than or equal to one make it a true statement. The inequality $x^2 + 1 < 0$ has no solution in **R** as no real value of x makes it a true statement.

To solve an inequality we can

(i) Add (or subtract) the same quantity to (from) both sides without changing the sign of inequality.

(ii) Multiply (or divide) both sides by the same positive quantity without changing the sign of inequality. However, if both sides of inequality are multiplied (or divided) by the same negative quantity the sign of inequality is reversed, i.e., '>' changes into '<' and vice versa.

6.1.3 *Representation of solution of linear inequality in one variable on a number line*

To represent the solution of a linear inequality in one variable on a number line, we use the following conventions:

(i) If the inequality involves '≥' or '≤', we draw filled circle (•) on the number line to indicate that the number corresponding to the filled circle is included in the solution set.

(ii) If the inequality involves '>' or '<', we draw an open circle (O) on the number line to indicate that the number corresponding to the open circle is excluded from the solution set.

6.1.4 *Graphical representation of the solution of a linear inequality*

(a) To represent the solution of a linear inequality in one or two variables graphically in a plane, we proceed as follows:

   (i) If the inequality involves '≥' or '≤', we draw the graph of the line as a thick line to indicate that the points on this line are included in the solution set.

   (ii) If the inequality involves '>' or '<', we draw the graph of the line as dotted line to indicate that the points on the line are excluded from the solution set.

(b) Solution of a linear inequality in one variable can be represented on number line as well as in the plane but the solution of a linear inequality in two variables of the type $ax + by > c$, $ax + by \geq c$, $ax + by < c$ or $ax + by \leq c$ $(a \neq 0,\ b \neq 0)$ can be represented in the plane only.

(c) Two or more inequalities taken together comprise a system of inequalities and the solutions of the system of inequalities are the solutions common to all the inequalities comprising the system.

6.1.5 *Two important results*

(a) If $a, b \in \mathbf{R}$ and $b \neq 0$, then

   (i) $ab > 0$ or $\dfrac{a}{b} > 0 \Rightarrow a$ and b are of the same sign.

   (ii) $ab < 0$ or $\dfrac{a}{b} < 0 \Rightarrow a$ and b are of opposite sign.

(b) If a is any positive real number, i.e., $a > 0$, then

   (i) $|x| < a \Leftrightarrow -a < x < a$

   $|x| \leq a \Leftrightarrow -a \leq x \leq a$

   (ii) $|x| > a \Leftrightarrow x < -a$ or $x > a$

   $|x| \geq a \Leftrightarrow x \leq -a$ or $x \geq a$

6.2 Solved Examples

Short Answer Type

Example 1 Solve the inequality, $3x - 5 < x + 7$, when

(i) x is a natural number (ii) x is a whole number

(iii) x is an integer (iv) x is a real number

Solution We have $3x - 5 < x + 7$

$\Rightarrow \quad 3x < x + 12$ (Adding 5 to both sides)

$\Rightarrow \quad 2x < 12$ (Subtracting x from both sides)

$\Rightarrow \quad x < 6$ (Dividing by 2 on both sides)

(i) Solution set is $\{1, 2, 3, 4, 5\}$

(ii) Solution set is $\{0, 1, 2, 3, 4, 5\}$

(iii) Solution set is $\{....-3, -2, -1, 0, 1, 2, 3, 4, 5\}$

(iv) Solution set is $\{x : x \in \mathbf{R} \text{ and } x < 6\}$, i.e., any real number less than 6.

Example 2 Solve $\dfrac{x-2}{x+5} > 2$

Solution We have $\dfrac{x-2}{x+5} > 2$

$\Rightarrow \quad \dfrac{x-2}{x+5} - 2 > 0$ [Subtracting 2 from each side]

$\Rightarrow \quad \dfrac{-(x+12)}{x+5} > 0$

$\Rightarrow \quad \dfrac{x+12}{x+5} < 0$ (Multiplying both sides by – 1)

$\Rightarrow \quad x + 12 > 0$ and $x + 5 < 0$ [Since $\dfrac{a}{b} < 0 \Rightarrow a$ and b are of opposite signs]

or

$x + 12 < 0$ and $x + 5 > 0$

$\Rightarrow \quad x > -12$ and $x < -5$

or

$x < -12$ and $x > -5$ (Not possible)

Therefore, $-12 < x < -5$, i.e. $x \in (-12, -5)$

Example 3 Solve $|3-4x| \geq 9$.

Solution We have $|3-4x| \geq 9$.

$\Rightarrow$ $3 - 4x \leq -9$ or $3 - 4x \geq 9$ (Since $|x| \geq a \Rightarrow x \leq -a$ or $x \geq a$)

$\Rightarrow$ $-4x \leq -12$ or $-4x \geq 6$

$\Rightarrow$ $x \geq 3$ or $x \leq \dfrac{-3}{2}$ (Dividing both sides by -4)

$\Rightarrow$ $x \in (-\infty, \dfrac{-3}{2}] \cup [3, \infty)$

Example 4 Solve $1 \leq |x-2| \leq 3$.

Solution We have $1 \leq |x-2| \leq 3$

$\Rightarrow$ $|x-2| \geq 1$ and $|x-2| \leq 3$

$\Rightarrow$ $(x - 2 \leq -1$ or $x - 2 \geq 1)$ and $(-3 \leq x - 2 \leq 3)$

$\Rightarrow$ $(x \leq 1$ or $x \geq 3)$ and $(-1 \leq x \leq 5)$

$\Rightarrow$ $x \in (-\infty, 1] \cup [3, \infty)$ and $x \in [-1, 5]$

Combining the solutions of two inequalities, we have

$x \in [-1, 1] \cup [3, 5]$

Example 5 The cost and revenue functions of a product are given by $C(x) = 20x + 4000$ and $R(x) = 60x + 2000$, respectively, where x is the number of items produced and sold. How many items must be sold to realise some profit?

Solution We have, profit = Revenue – Cost

$= (60x + 2000) - (20x + 4000)$

$= 40x - 2000$

To earn some profit, $40x - 2000 > 0$

$\Rightarrow$ $x > 50$

Hence, the manufacturer must sell more than 50 items to realise some profit.

Example 6 Solve for x, $|x+1| + |x| > 3$.

Solution On LHS of the given inequality, we have two terms both containing modulus. By equating the expression within the modulus to zero, we get $x = -1, 0$ as critical points. These critical points divide the real line in three parts as $(-\infty, -1)$, $[-1, 0)$, $[0, \infty)$.

Case I When $-\infty < x < -1$

$$|x+1| + |x| > 3 \Rightarrow -x-1-x > 3 \Rightarrow x < -2.$$

Case II When $-1 \le x < 0$,

$$|x+1| + |x| > 3 \Rightarrow x+1-x > 3 \Rightarrow 1 > 3 \qquad \text{(not possible)}$$

Case III When $0 \le x < \infty$,

$$|x+1| + |x| > 3 \Rightarrow x+1+x > 3 \Rightarrow x > 1.$$

Combining the results of cases (I), (II) and (III), we get

$$x \in (-\infty, -2) \cup (1, \infty)$$

Long Answer Type

Example 7 Solve for x, $\dfrac{|x+3| + x}{x+2} > 1$

Solution We have $\dfrac{|x+3| + x}{x+2} > 1$

$$\Rightarrow \qquad \frac{|x+3| + x}{x+2} - 1 > 0$$

$$\Rightarrow \qquad \frac{|x+3| - 2}{x+2} > 0$$

Now two cases arise:

Case I When $x + 3 \ge 0$, i.e., $x \ge -3$. Then

$$\frac{|x+3| - 2}{x+2} > 0 \Rightarrow \frac{x+3-2}{x+2} > 0$$

$$\Rightarrow \qquad \frac{x+1}{x+2} > 0$$

$$\Rightarrow \qquad \{(x+1) > 0 \text{ and } x+2 > 0\} \quad \text{or} \quad \{x+1 < 0 \text{ and } x+2 < 0\}$$

$$\Rightarrow \qquad \{x > -1 \text{ and } x > -2\} \quad \text{or} \quad \{x < -1 \text{ and } x < -2\}$$

$$\Rightarrow \qquad x > -1 \quad \text{or} \quad x < -2$$

$$\Rightarrow \qquad x \in (-1, \infty) \quad \text{or} \quad x \in (-\infty, -2)$$

$$\Rightarrow \qquad x \in (-3, -2) \cup (-1, \infty) \quad [\text{Since } x \ge -3] \qquad \ldots (1)$$

Case II When $x + 3 < 0$, i.e., $x < -3$

$$\frac{|x+3|-2}{x+2} > 0 \quad \Rightarrow \quad \frac{-x-3-2}{x+2} > 0$$

$$\Rightarrow \quad \frac{-(x+5)}{x+2} > 0 \quad \Rightarrow \quad \frac{x+5}{x+2} < 0$$

$\Rightarrow$ $(x + 5 < 0 \text{ and } x + 2 > 0)$ or $(x + 5 > 0 \text{ and } x + 2 < 0)$

$\Rightarrow$ $(x < -5 \text{ and } x > -2)$ or $(x > -5 \text{ and } x < -2)$

it is not possible.

$\Rightarrow$ $x \in (-5, -2)$ ... (2)

Combining (I) and (II), the required solution is

$$x \in (-5, -2) \cup (-1, \infty)$$

Example 8 Solve the following system of inequalities :

$$\frac{x}{2x+1} \geq \frac{1}{4}, \quad \frac{6x}{4x-1} < \frac{1}{2}$$

Solution From the first inequality, we have $\frac{x}{2x+1} - \frac{1}{4} \geq 0$

$\Rightarrow$ $\frac{2x-1}{2x+1} \geq 0$

$\Rightarrow$ $(2x - 1 \geq 0 \text{ and } 2x + 1 > 0)$ or $(2x - 1 \leq 0 \text{ and } 2x + 1 < 0)$ [Since $2x + 1 \neq 0$)

$\Rightarrow$ $(x \geq \frac{1}{2} \text{ and } x > -\frac{1}{2})$ or $(x \leq \frac{1}{2} \text{ and } x < -\frac{1}{2})$

$\Rightarrow$ $x \geq \frac{1}{2}$ or $x < -\frac{1}{2}$

$\Rightarrow$ $x \in (-\infty, -\frac{1}{2}) \cup [\frac{1}{2}, \infty)$ ... (1)

From the second inequality, we have $\frac{6x}{4x-1} - \frac{1}{2} < 0$

$\Rightarrow$ $\frac{8x+1}{4x-1} < 0$

$\Rightarrow$ $(8x + 1 < 0 \text{ and } 4x - 1 > 0)$ or $(8x + 1 > 0 \text{ and } 4x - 1 < 0)$

$\Rightarrow$ $(x < -\frac{1}{8} \text{ and } x > \frac{1}{4})$ or $(x > -\frac{1}{8} \text{ and } x < \frac{1}{4})$

$\Rightarrow$ $x \in (-\frac{1}{8}, \frac{1}{4})$ (Since the first is not possible) ... (2)

Note that the common solution of (1) and (2) is null set. Hence, the given system of inequalities has no solution.

Example 9 Find the linear inequalities for which the shaded region in the given figure is the solution set.

Solution

(i) Consider $2x + 3y = 3$. We observe that the shaded region and the origin lie on opposite side of this line and (0, 0) satisfies $2x + 3y \leq 3$. Therefore, we must have $2x + 3y \geq 3$ as linear inequality corresponding to the line $2x + 3y = 3$.

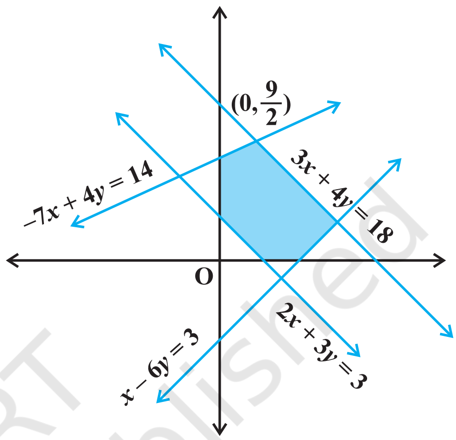

Fig 6.1

(ii) Consider $3x + 4y = 18$. We observe that the shaded region and the origin lie on the same side of this line and (0, 0) satisfies $3x + 4y \leq 18$. Therefore, $3x + 4y \leq 18$ is the linear inequality corresponding to the line $3x + 4y = 18$.

(iii) Consider $-7x + 4y = 14$. It is clear from the figure that the shaded region and the origin lie on the same side of this line and (0, 0) satisfies the inequality $-7x + 4y \leq 14$. Therefore, $-7x + 4y \leq 14$ is the inequality corresponding to the line $-7x + 4y = 14$.

(iv) Consider $x - 6y = 3$. It may be noted that the shaded portion and origin lie on the same side of this line and (0, 0) satisfies $x - 6y \leq 3$. Therefore, $x - 6y \leq 3$ is the inequality corresponding to the line $x - 6y = 3$.

(v) Also the shaded region lies in the first quadrant only. Therefore, $x \geq 0$, $y \geq 0$.

Hence, in view of (i), (ii), (iii), (iv) and (v) above, the linear inequalities corresponding to the given solution set are :

$2x + 3y \geq 3, 3x + 4y \leq 18$ $-7x + 4y \leq 14$, $x - 6y \leq 3$, $x \geq 0$, $y \geq 0$.

Objective Type

Choose the correct answer from the given four options against each of the Examples 10 to 13 (M.C.Q.)

Example 10 If $\frac{|x-2|}{x-2} \geq 0$, then

(A) $x \in [2, \infty)$ (B) $x \in (2, \infty)$ (C) $x \in (-\infty, 2)$ (D) $x \in (-\infty, 2]$

Solution (B) is the correct choice. Since $\dfrac{|x-2|}{x-2} \geq 0$, for $|x-2| \geq 0$, and $x - 2 \neq 0$.

Example 11 The length of a rectangle is three times the breadth. If the minimum perimeter of the rectangle is 160 cm, then

(A) breadth > 20 cm (B) length < 20 cm

(C) breadth $x \geq 20$ cm (D) length ≤ 20 cm

Solution (C) is the correct choice. If x cm is the breadth, then

$$2\,(3x + x) \geq 160 \Rightarrow x \geq 20$$

Example 12 Solutions of the inequalities comprising a system in variable x are represented on number lines as given below, then

Fig 6.2

(A) $x \in (-\infty, -4] \cup [3, \infty)$ (B) $x \in [-3, 1]$

(C) $x \in (-\infty, -4) \cup [3, \infty)$ (D) $x \in [-4, 3]$

Solution (A) is the correct choice

Common solution of the inequalities is from $-\infty$ to -4 and 3 to ∞.

Example 13 If $|x+3| \geq 10$, then

(A) $x \in (-13, 7]$ (B) $x \in (-13, 7]$

(C) $x \in (-\infty, -13] \cup [7, \infty)$ (D) $x \in [-\infty, -13] \cup [7, \infty)$

Solution (D) is the correct choice, since $|x+3| \geq 10$, $\Rightarrow x + 3 \leq -10$ or $x + 3 \geq 10$

$\Rightarrow$ $x \leq -13$ or $x \geq 7$

$\Rightarrow$ $x \in (-\infty, -13] \cup [7, \infty)$

Example 14 State whether the following statements are True or False.

(i) If $x > y$ and $b < 0$, then $bx < by$

(ii) If $xy > 0$, then $x > 0$, and $y < 0$

(iii) If $xy < 0$, then $x > 0$, and $y > 0$

(iv) If $x > 5$ and $x > 2$, then $x \in (5, \infty)$

(v) If $|x| < 5$, then $x \in (-5, 5)$

(vi) Graph of $x > -2$ is

(vii) Solution set of $x - y \leq 0$ is

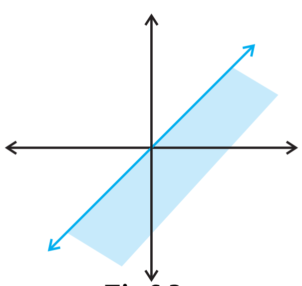

Fig 6.3

Solution

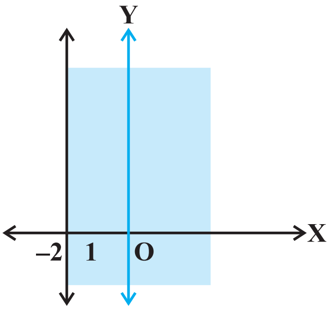

Fig 6.4

(i) True, because the sign of inequality is reversed when we multiply both sides of an inequality by a negative quantity.

(ii) False, product of two numbers is positive if they have the same sign.

(iii) False, product of two numbers is negative if they have opposite signs.

(iv) True

(v) True if $|x| < 5 \Rightarrow -5 < x < 5 \Rightarrow x \in (-5, 5)$.

(vi) False, because for $x > -2$, the line $x = -2$ has to be dotted, i.e., the region does not include the points on the line $x = -2$

(vii) False, because (1, 0) does not satisfy the given inequality and it is a point in shaded portion.

Example 15 Fill in the blanks in the following:

(i) If $x \geq -3$, then $x + 5$ 2

(ii) If $-x \leq -4$, then $2x$ 8

(iii) If $\dfrac{1}{x-2} < 0$, then x 2

(iv) If $a < b$ and $c < 0$, then $\dfrac{a}{c}$............$\dfrac{b}{c}$

(v) If $|x-1| \leq 2$, then -1..... x 3

(vi) If $|3x - 7| > 2$, then x .... $\dfrac{5}{3}$ or x 3

(vii) If $p > 0$ and $q < 0$, then $p + q$ ... p

Solution

(i) ($\geq$), because same number can be added to both sides of inequality without changing the sign of inequality.

(ii) ($\geq$), after multiplying both sides by – 2, the sign of inequality is reversed.

(iii) (<), because if $\dfrac{a}{b} < 0$ and $a > 0$, then $b < 0$.

(iv) (>), if both sides are divided by the same negative quantity, then the sign of inequality is reversed.

(v) $(\leq, \leq)$, $|x-1| \leq 2 \Rightarrow -2 \leq x-1 \leq 2 \Rightarrow -1 \leq x \leq 3.$

(vi) $(<, >)$, $|3x-7| > 2 \Rightarrow 3x-7 < -2$ or $3x-7 > 2$

$\Rightarrow x < \frac{5}{3}$ or $x > 3$

(vii) $(<)$, as p is positive and q is negative, therefore, $p + q$ is always smaller than p.

6.3 EXERCISE

Short Answer Type

Solve for x, the inequalities in Exercises 1 to 12.

1. $\frac{4}{x+1} \leq 3 \leq \frac{6}{x+1}$, $(x > 0)$ **2.** $\frac{|x-2|-1}{|x-2|-2} \leq 0$ **3.** $\frac{1}{|x|-3} \leq \frac{1}{2}$

4. $|x-1| \leq 5$, $|x| \geq 2$ **5.** $-5 \leq \frac{2-3x}{4} \leq 9$

6. $4x + 3 \geq 2x + 17$, $3x - 5 < -2$.

7. A company manufactures cassettes. Its cost and revenue functions are $C(x) = 26{,}000 + 30x$ and $R(x) = 43x$, respectively, where x is the number of cassettes produced and sold in a week. How many cassettes must be sold by the company to realise some profit?

8. The water acidity in a pool is considerd normal when the average pH reading of three daily measurements is between 8.2 and 8.5. If the first two pH readings are 8.48 and 8.35, find the range of pH value for the third reading that will result in the acidity level being normal.

9. A solution of 9% acid is to be diluted by adding 3% acid solution to it. The resulting mixture is to be more than 5% but less than 7% acid. If there is 460 litres of the 9% solution, how many litres of 3% solution will have to be added?

10. A solution is to be kept between 40°C and 45°C. What is the range of temperature in degree fahrenheit, if the conversion formula is $F = \frac{9}{5}C + 32$?

11. The longest side of a triangle is twice the shortest side and the third side is 2cm longer than the shortest side. If the perimeter of the triangle is more than 166 cm then find the minimum length of the shortest side.

12. In drilling world's deepest hole it was found that the temperature T in degree celcius, x km below the earth's surface was given by $T = 30 + 25(x - 3)$, $3 \leq x \leq 15$. At what depth will the temperature be between 155°C and 205°C?

Long Answer Type

13. Solve the following system of inequalities $\frac{2x+1}{7x-1} > 5$, $\frac{x+7}{x-8} > 2$

14. Find the linear inequalities for which the shaded region in the given figure is the solution set.

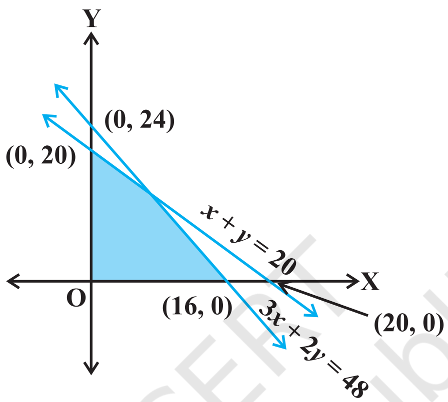

Fig 6.5

15. Find the linear inequalities for which the shaded region in the given figure is the solution set.

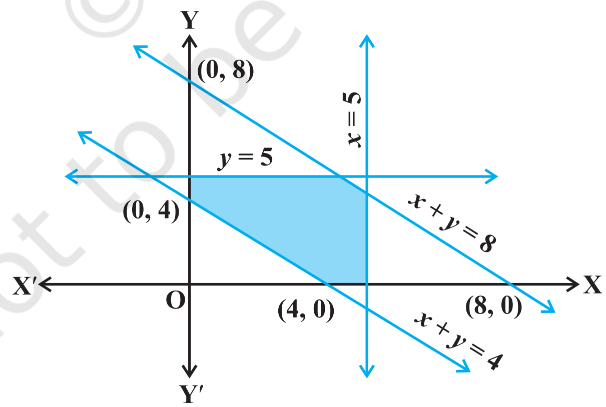

Fig 6.6

16. Show that the following system of linear inequalities has no solution

$x + 2y \leq 3$, $3x + 4y \geq 12$, $x \geq 0$, $y \geq 1$

17. Solve the following system of linear inequalities:

$3x + 2y \geq 24$, $3x + y \leq 15$, $x \geq 4$

18. Show that the solution set of the following system of linear inequalities is an unbounded region

$2x + y \geq 8$, $x + 2y \geq 10$, $x \geq 0$, $y \geq 0$

Objective Type Question

Choose the correct answer from the given four options in each of the Exercises 19 to 26 (M.C.Q.).

19. If $x < 5$, then

(A) $-x < -5$ (B) $-x \leq -5$

(C) $-x > -5$ (D) $-x \geq -5$

20. Given that x, y and b are real numbers and $x < y$, $b < 0$, then

(A) $\frac{x}{b} < \frac{y}{b}$ (B) $\frac{x}{b} \leq \frac{y}{b}$

(C) $\frac{x}{b} > \frac{y}{b}$ (D) $\frac{x}{b} \geq \frac{y}{b}$

21. If $-3x + 17 < -13$, then

(A) $x \in (10, \infty)$ (B) $x \in [10, \infty)$

(C) $x \in (-\infty, 10]$ (D) $x \in [-10, 10)$

22. If x is a real number and $|x| < 3$, then

(A) $x \geq 3$ (B) $-3 < x < 3$

(C) $x \leq -3$ (D) $-3 \leq x \leq 3$

23. x and b are real numbers. If $b > 0$ and $|x| > b$, then

(A) $x \in (-b, \infty)$ (B) $x \in [-\infty, b)$

(C) $x \in (-b, b)$ (D) $x \in (-\infty, -b) \cup (b, \infty)$

24. If $|x-1| > 5$, then

(A) $x \in (-4, 6)$ (B) $x \in [-4, 6]$

(C) $x \in [-\infty, -4) \cup (6, \infty)$ (D) $x \in [-\infty, -4) \cup [6, \infty)$

25. If $|x+2| \leq 9$, then

(A) $x \in (-7, 11)$ (B) $x \in [-11, 7]$

(C) $x \in (-\infty, -7) \cup (11, \infty)$ (D) $x \in (-\infty, -7) \cup [11, \infty)$

26. The inequality representing the following graph is:

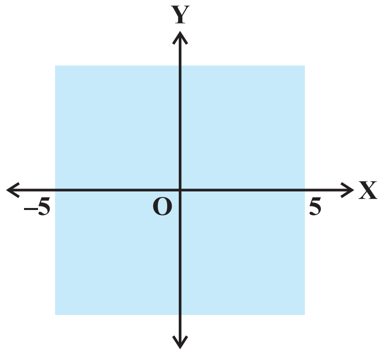

Fig 6.7

(A) $|x| < 5$ (B) $|x| \le 5$ (C) $|x| > 5$ (D) $|x| \ge 5$

Solution of a linear inequality in variable x is represented on number line in Exercises 27 to 30. Choose the correct answer from the given four options in each of the exercises (M.C.Q.).

27. (A) $x \in (-\infty, 5)$ (B) $x \in (-\infty, 5]$

(C) $x \in [5, \infty,)$ (D) $x \in (5, \infty)$

Fig 6.8

28. (A) $x \in (\frac{9}{2}, \infty)$

(B) $x \in [\frac{9}{2}, \infty)$

(D) $x \in [-\infty, \frac{9}{2})$

(D) $x \in (-\infty, \frac{9}{2}]$

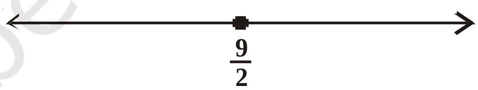

Fig 6.9

29. (A) $x \in (-\infty, \frac{7}{2})$ (B) $x \in (-\infty, \frac{7}{2}]$

(C) $x \in [\frac{7}{2}, -\infty)$ (D) $x \in (\frac{7}{2}, \infty)$

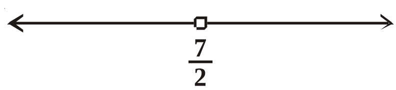

Fig 6.10

30. (A) $x \in (-\infty, -2)$

(B) $x \in (-\infty, -2]$

(C) $x \in (-2, \infty]$

(D) $x \in [-2, \infty)$

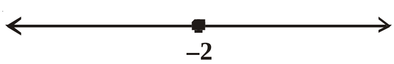

Fig 6.11

31. State which of the following statements is True or False

(i) If $x < y$ and $b < 0$, then $\frac{x}{b} < \frac{y}{b}$.

(ii) If $xy > 0$, then $x > 0$ and $y < 0$

(iii) If $xy > 0$, then $x < 0$ and $y < 0$

(iv) If $xy < 0$, then $x < 0$ and $y < 0$

(v) If $x < -5$ and $x < -2$, then $x \in (-\infty, -5)$

(vi) If $x < -5$ and $x > 2$, then $x \in (-5, 2)$

(vii) If $x > -2$ and $x < 9$, then $x \in (-2, 9)$

(viii) If $|x| > 5$, then $x \in (-\infty, -5) \cup [5, \infty)$

(ix) If $|x| \leq 4$, then $x \in [-4, 4]$

(x) Graph of $x < 3$ is

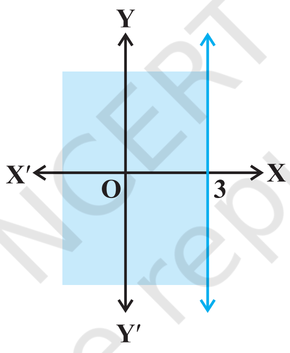

Fig 6.12

(xi) Graph of $x \geq 0$ is

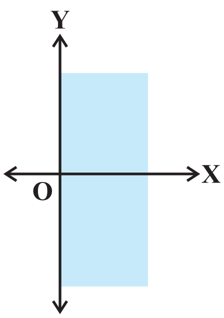

Fig 6.13

(xii) Graph of $y \leq 0$ is

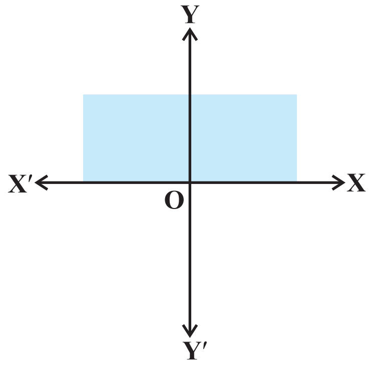

Fig 6.14

(xiii) Solution set of $x \geq 0$ and $y \leq 0$ is

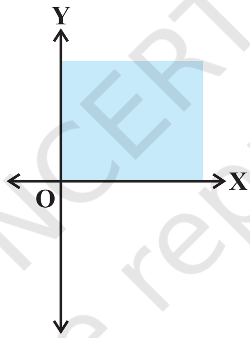

Fig 6.15

(xiv) Solution set of $x \geq 0$ and $y \leq 1$ is

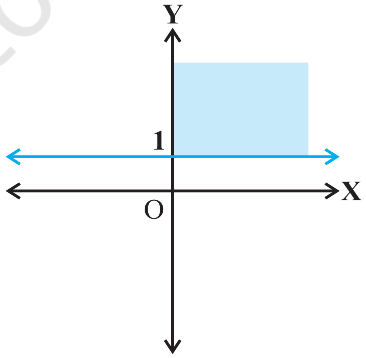

Fig 6.16

(xv) Solution set of $x + y \geq 0$ is

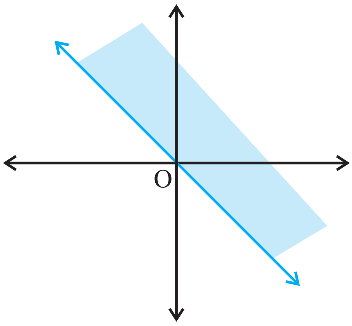

Fig 6.17

32. Fill in the blanks of the following:

(i) If $-4x \geq 12$, then x ... -3.

(ii) If $\frac{-3}{4}x \leq -3$, then x ... 4.

(iii) If $\frac{2}{x+2} > 0$, then x ... -2.

(iv) If $x > -5$, then $4x$ ... -20.

(v) If $x > y$ and $z < 0$, then $-xz$ ... $-yz$.

(vi) If $p > 0$ and $q < 0$, then $p - q$ ... p.

(vii) If $|x+2| > 5$, then x ... -7 or x ... 3.

(viii) If $-2x + 1 \geq 9$, then x ... -4.